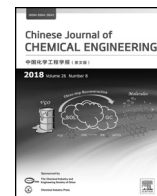




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Article

Synthesis of refrigeration system based on generalized disjunctive programming model☆

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ABSTRACT

Refrigeration system holds an important role in process industries. The optimal synthesis cannot only reduce the energy consumption, but also save the production costs. In this study, a general methodology is developed for the optimal design of refrigeration cycle and heat exchanger network (HEN) simultaneously. Taking the heat integration between the external heat sources/sinks and the refrigeration cycle into consideration, a superstructure with sub-coolers is developed. Through defining logical variables that indicate the relative temperature positions of refrigerant streams after sub-coolers, the synthesis is formulated as a Generalized Disjunctive Programming (GDP) problem based on LP transshipment model, with the target of minimizing the total compressor shaft work in the refrigeration system. The GDP model is then reformulated as a Mixed Integer Nonlinear Programming (MINLP) problem with the aid of binary variables and Big-M Constraint Method. The efficacy of the process synthesis model is demonstrated by a case study of ethylene refrigeration system. The result shows that the optimization can significantly reduce the exergy loss as well as the total compression shaft work.

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1. Introduction

Refrigeration system is widely used to provide cold utilities for petrochemical processes. So it holds an important role in process industries. Generally speaking, it works by obtaining heats from low-temperature sources and passing them to high-temperature sinks, with the compression work consumed. The Fig. 1 shows a diagram of a simple refrigeration loop, composed of four components: condenser, expansion valve, evaporator and compressor. The liquid refrigerant gets heat from a process stream in the evaporator section and delivers it to other process streams through the condenser. In this cycle, the refrigerant can be repeatedly compressed, condensed, expanded and evaporated to achieve the cooling requirement continuously. In addition, multiple pressure levels can simultaneously exist in the refrigeration system, according to the specific requirements. The optimization of the refrigeration system, which is a large energy consumer, can reduce the energy loss and further save the final production cost. Therefore, this issue has been widely studied in decades.

Barnés and King [1] developed one of the earlier models of a refrigeration system. They used heuristics and dynamic programming simultaneously to increase intermediate pressure stages. Cheng and Math [2] improved the model, and extended the system to mixed refrigerant cycle. Because of the significant dependence on heuristics, these methods could not guarantee the solution optimality for complicated refrigeration system. Under this circumstance, the mathematical programming method was used by Shelton and Grossmann [3], who put forward a mixed-integer linear programming (MILP) method based on the superstructure of the refrigeration system. From then on, a large number of studies were carried out on this foundation and more reasonable superstructures were established [4–7]. However, the research on the optimization of refrigeration system with multi-temperature levels was still lacking.

On this basis, Zhang and Xu [8] proposed a general methodology for the optimal process synthesis of a cascade refrigeration system (CRS) with multiple temperature levels and multiple refrigerants, based on exergy analysis. Luo, Feng, Sun and Yuan [9] presented a systematic approach for optimizing low-temperature system through mathematical method and exergy analysis. Then, Zhang, Xu and Dinh [10] simplified the superstructure of CRS and presented an MILP method. They assumed that the refrigerant from level m'' to level m flowed through either a sub-cooler or an expansion valve. However, the possibility of refrigerant passing through an expansion valve after a sub-cooler was ignored. Hence, the simplified model was impractical

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According to the superstructure shown in Fig. 2, an MINLP model including material and energy balance of each unit operation with equality and inequality constraints by considering all possibilities will be developed and solved.

2.2. MINLP model development

2.2.1. Mass balance equations

The MINLP model consists of five parts, i.e. mass balance, energy balance, equality constraints, inequality constraints and objective function. The GDP model is used because of the branching selections existing in the HEN transshipment model.

The mass balance of the compressors is represented by Eq. (1):

$$fg_m + \sum_{m'=m+1}^M fc_{m',m} = \sum_{m'=0}^{m-1} fc_{m,m'}, \quad \forall m \in SM \quad (1)$$

where fg_m is the mass flow rate of the saturated vapor from flash drum of level m ; $fc_{m',m}$ is the mass flow rate of the stream into the compressor connecting the level m' and m ; $fc_{m,m'}$ is the mass flow rate of the stream out of the compressor connecting the level m and m' . Note that the set $SM = \{m|0, \dots, M\}$ is the combination of all possible pressure levels; level m'' is upper than m while m' is the lower one.

The refrigerant flow from a higher-pressure level to a lower one passes through the sub-coolers and expansion valves. Mass balance for all possible scenarios about whether a sub-cooler exists or not between any two levels is shown in the Eqs. (2) and (3):

$$fx_m = \sum_{m'=0}^{m-1} fsc_{m',m}, \quad \forall m \in SM, m < M \quad (2)$$

$$fe_m = \sum_{m'=m+1}^M fsc_{m,m'}, \quad \forall m \in SM, m < M \quad (3)$$

where fx_m is the inlet mass flow rate of flash drum at level m ; $fsc_{m',m}$ and $fsc_{m,m'}$ are mass flow rates of refrigerant through the sub-coolers or/and expansion valves entering and leaving level m ; fe_m is the mass flow rate of the saturated liquid from flash drum of level m .

If the expansion valve exists between the two chosen levels, fv_m , the amount of vapor generated by the valve at m^{th} pressure level, can be calculated as follows:

$$fv_m = \sum_{m'=0}^{m-1} fsc_{m',m} vf_{m',m}, \quad \forall m \in SM \quad (4)$$

where $vf_{m',m}$ is the vapor fraction of refrigerant leaving the valve. As mentioned earlier, the refrigerant flows through the valve under the isenthalpic condition, so according to the mass balance and energy balance, the $vf_{m',m}$ can be calculated as follows:

$$vf_{m',m} = \frac{HF_{m',m} - HL_m}{HV_m - HL_m}, \quad \forall m, m' \in SM, m > m', m > 0 \quad (5)$$

where $HF_{m',m}$ is the mass specific enthalpy of the stream at the entrance of the expansion valve. Meanwhile, HV_m and HL_m indicate gas specific enthalpy and liquid specific enthalpy at the outlet of the expansion respectively, as shown in Fig. 3.

$$HF_{m',m} = 2.5681T_{sc_{m',m}} + 1479.5, \quad \forall m, m' \in SM, m > m', m > 0 \quad (6)$$

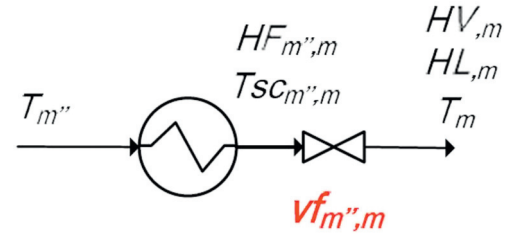


Fig. 3. The sketch of the sub-cooler and valve section.

$HF_{m'',m}$ can be expressed by its corresponding $T_{sc_{m'',m}}$, which is the temperature of the refrigerant leaving the sub-cooler from level m'' to level m . Here, the values of HV_m and HL_m can be obtained from the thermodynamic database.

Eqs. (7) and (8) represent mass balance of the saturated vapor and liquid streams leaving the flash drum at level m , where fv_m is the mass flow rate of saturated vapor generated by the evaporating section at m^{th} level.

$$fg_m = fve_m + fv_m, \quad \forall m \in SM, m > 0 \quad (7)$$

$$fe_m = fx_m - fg_m, \quad \forall m \in SM, m > 0 \quad (8)$$

2.2.2. Energy balance equations

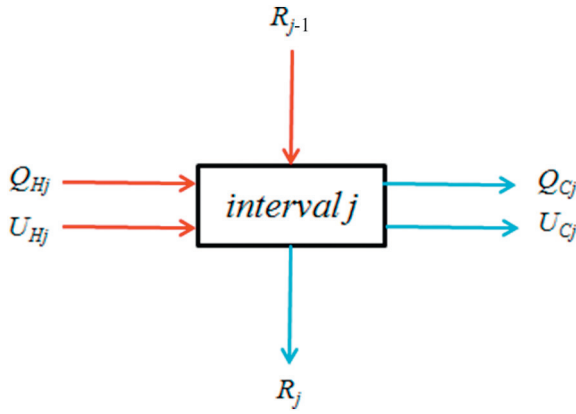
In the low temperature separation system, the heat exchanger network and the refrigeration system are strongly interacted. The heat exchanger network determines the minimum demand for cold utility. However, the levels of refrigeration system and cooling amount generated by the refrigerant directly affect the design of HEN. Therefore, the synthesis of the refrigeration system must take the design of HEN into consideration to obtain the optimal structure. In order to solve this problem, this paper uses the HEN synthesis transshipment model to calculate the amount of refrigerant in the refrigeration system. Combining transport model with the refrigeration system superstructure, the minimum shaft work of the compressors, and the simple structure of HEN and refrigeration system could be obtained finally.

In a multi-stage refrigeration system, the cold amount required for the condenser at the 0th level is generally provided by other refrigeration systems and is not under consideration in this model.

To construct the HEN synthesis transshipment model, firstly the temperature intervals are divided, which are expressed as $S_j = \{j|0, \dots, J\}$. The temperature ranges include the following three cases: (1) the inlet temperatures of all the process streams; (2) the maximum or minimum outlet temperatures (3) the inlet temperatures of the utility flows. In addition, the minimum temperature approach in the HEN is 2 °C. Besides, for the sake of calculation, the temperature rise of the isothermal evaporator is set to 0.2 °C. Then, according to the HEN synthesis transshipment model proposed by Papoulias and Grossmann [11], for each heat interval, as shown in Fig. 4, the heat balance relationship of the j^{th} temperature interval is established in Eq. (9).

$$R_j - R_{j-1} - U_{Hj} + U_{Cj} = Q_{Hj} - Q_{Cj} \quad (9)$$

In this equation, R_{j-1} and R_j are the remaining heat at the $(j-1)^{\text{th}}$ and j^{th} temperature intervals, respectively. U_{Hj} and U_{Cj} are the heat provided or removed by the hot and cold utilities, respectively. Similarly, Q_{Hj} and Q_{Cj} are the heat provided or removed by the process streams.

Fig. 4. Diagrammatic sketch of the heat exchange in j^{th} interval.

According to the superstructure, the HEN synthesis transshipment model can be expressed as follows:

$$R_j - R_{j-1} - \sum_{m'=0}^{m-1} \sum_{m=1}^M (Q_{t_{j,m'},m} z_{j,m',m}) + \sum_{m=1}^M (Q_{x_m} x_{i,j,m}) = \sum_{n=1}^N Q_{h_{j,n}} - \sum_{l=1}^L Q_{c_{j,l}} \quad \forall j \in SJ \quad (10)$$

Here, R_j and R_{j-1} have the same meanings as in Eq. (7). $Q_{t_{j,m'},m}$, whose detailed calculation method will be introduced in Section 2.2.3, means the heat duty from sub-coolers of m'' -level to the m -level, in temperature interval j ; and Q_{x_m} indicates the duty of the cold side offered by evaporators at the m -level for the temperature corresponding interval. $z_{j,m',m}$ and $x_{i,j,m}$ are both binary variables, which determine whether the temperature of sub-cooling sections and the evaporating sections cross the j -th interval. Variables will be given as known quantities in specific problem, with the exception of R_{j-1} , R_j , $Q_{t_{j,m'},m}$ and Q_{x_m} . It should be noted that Q_{x_m} can be calculated by the mass flow rate of vapor produced at the m -th evaporator, which is represented as fve_m , and the latent heat of vaporization of refrigerant under the m -th pressure, shown as Δhr_m . Under the hypothesis of ideal gas, Δhr_m can be obtained from the thermodynamic database.

$$Q_{x_m} = fve_m \Delta hr_m \quad \forall m \in SM, m > 0 \quad (11)$$

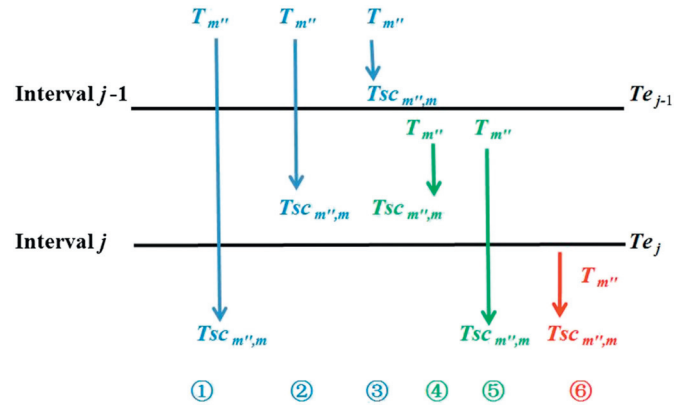
2.2.3. Inequality constraints based on GDP method

Based on the super structure proposed in Fig. 2, the refrigerant from level m'' passes through the expansion valve after the sub-cooler to level m . The outlet temperature of the sub-cooler is a continuous variable, which can be represented as $T_{sc_{m',m}}$. The heat duty $Q_{t_{j,m'},m}$ can be regarded as a segmentation function with respect to $T_{sc_{m',m}}$. Hence, the GDP model [12–15] is introduced to solve the branching selections. The relative position of the inlet and outlet temperatures of sub-coolers to temperature interval j is shown in Fig. 5, where $T_{m''}$ represents temperature of refrigerant before the sub-cooler from level m'' to level m , and $T_{sc_{m',m}}$ indicates the outlet temperature; Te_{j-1} and Te_j are the upper and lower boundaries of the interval j , respectively. Besides, $T_{sc_{m',m}}$ is a continuous variable ranging from T_m to $T_{m''}$.

$$T_{sc_{m',m}} \leq T_{m''} \quad \forall m, m'' \in SM, m > m'', m > 0 \quad (12)$$

$$T_{sc_{m',m}} \geq T_m \quad \forall m, m'' \in SM, m > m'', m > 0 \quad (13)$$

There are six position relationships in Fig. 5. Since the values of $T_{m''}$, Te_{j-1} and Te_j are known, the last three can be represented by the binary

Fig. 5. Relative position of streams in sub-cooling section to temperature j .

variables $z_{j,m',m}$. Based on this, the GDP model can be established. In order to show the relative position relationships, the logical variables are defined as below:

- Y_j^1 If $T_{sc_{m',m}}$ is smaller than Te_j , Y_j^1 is True.
- Y_j^2 If $T_{sc_{m',m}}$ is between Te_j and Te_{j-1} , Y_j^2 is True.
- Y_j^3 If $T_{sc_{m',m}}$ is bigger than Te_{j-1} , Y_j^3 is True.

GDP expressions are as follows:

$$\left[\begin{array}{l} Y_j^1 \\ T_{sc_{m',m}} \leq Te_j \\ Q_{t_{j,m'},m} = fsc_{m',m} C_p (T_{j-1} - T_j) \end{array} \right] \vee \left[\begin{array}{l} Y_j^2 \\ T_j \leq T_{sc_{m',m}} \leq Te_{j-1} \\ Q_{t_{j,m'},m} = fsc_{m',m} C_p (T_{j-1} - T_{sc_{m',m}}) \end{array} \right] \vee \left[\begin{array}{l} Y_j^3 \\ T_{sc_{m',m}} \geq Te_{j-1} \\ Q_{t_{j,m'},m} = 0 \end{array} \right] \quad \forall j \in SJ \quad (14)$$

$$Y_j^1, Y_j^2, Y_j^3 \in \{True, False\} \quad (15)$$

There are three constraints in the equation, and only one constraint will take effect, while the rest do not work. Each logical variable should satisfy the constraint (16):

$$Y_j^1 \vee Y_j^2 \vee Y_j^3 \quad (16)$$

Through the binary variables, the GDP expression can be reformulated into MINLP model by means of the Big-M method [16]. The equality constraints in GDP can be replaced by two inequality constraints.

$$y_{j,m',m}^1, y_{j,m',m}^2, y_{j,m',m}^3 \in \{0, 1\} \quad (17)$$

- (a) When $T_{sc_{m',m}}$ is smaller than Te_j , Y_j^1 is True, so $y_{j,m',m}^1$ equals one. The heat quantity provided by refrigerants at sub-cooling section can be calculated by the inequality equations. $y_{j,m',m}^1, y_{j,m',m}^2$ and $y_{j,m',m}^3$ are binary variables, and have the same meanings with the relative logical variables Y_j^1, Y_j^2 and Y_j^3 . M_1 and M_2 are large-enough numbers.

$$\left. \begin{array}{l} T_{sc_{m',m}} \leq Te_j + M_1 (1 - y_{j,m',m}^1) \\ Q_{t_{j,m'},m} \geq fsc_{m',m} C_p (Te_{j-1} - Te_j) - M_2 (1 - y_{j,m',m}^1) \\ Q_{t_{j,m'},m} \leq fsc_{m',m} C_p (Te_{j-1} - Te_j) + M_2 (1 - y_{j,m',m}^1) \end{array} \right\} \quad \forall j \in SJ \quad (18)$$

- (b) When $T_{sc_{m',m}}$ is between Te_j and Te_{j-1} , Y_j^2 is True, so $y_{j,m',m}^2$ equals one. Then $Qt_{j,m',m}$ can be expressed by constraints (19):

$$\left. \begin{aligned} T_{sc_{m',m}} &\leq Te_{j-1} + M_1(1 - y_{j,m',m}^2) \\ T_{sc_{m',m}} &\geq Te_j - M_1(1 - y_{j,m',m}^2) \\ Qt_{j,m',m}^2 &\geq f_{sc_{m',m}} C_p (Te_{j-1} - T_{sc_{m',m}}) - M_2(1 - y_{j,m',m}^2) \\ Qt_{j,m',m}^2 &\leq f_{sc_{m',m}} C_p (Te_j - T_{sc_{m',m}}) + M_2(1 - y_{j,m',m}^2) \end{aligned} \right\} \quad \forall j \in SJ \quad (19)$$

- (c) When $T_{sc_{m',m}}$ is bigger than Te_{j-1} , Y_j^3 is True, so $y_{j,m',m}^3$ equals one, and the inequalities change to constraints (20):

$$\left. \begin{aligned} T_{sc_{m',m}} &\geq Te_{j-1} - M_1(1 - y_{j,m',m}^3) \\ Qt_{j,m',m}^3 &\leq M_2(1 - y_{j,m',m}^3) \end{aligned} \right\} \quad \forall j \in SJ \quad (20)$$

Only one of the three sets of constraints will come into force in interval j . Besides, when $y_{j,m',m}$ is equal to zero, the relative $Qt_{j,m',m}$ also equals zero. Therefore, the logical constraints (21)–(24) are added:

$$y_{j,m',m}^1 + y_{j,m',m}^2 + y_{j,m',m}^3 = 1 \quad (21)$$

$$Qt_{j,m',m}^1 \leq M_2 y_{j,m',m}^1 \quad (22)$$

$$Qt_{j,m',m}^2 \leq M_2 y_{j,m',m}^2 \quad (23)$$

$$Qt_{j,m',m}^3 \leq M_2 y_{j,m',m}^3 \quad (24)$$

$$Qt_{j,m',m}^1 \geq 0, \quad Qt_{j,m',m}^2 \geq 0, \quad Qt_{j,m',m}^3 \geq 0 \quad (25)$$

Accordingly, heat provided by refrigerants through sub-coolers from level m'' to level m can be calculated by Eq. (26).

$$Qt_{j,m',m} = Qt_{j,m',m}^1 + Qt_{j,m',m}^2 + Qt_{j,m',m}^3 \quad (26)$$

2.2.4. Objective function

In this paper, the target of the mathematical model is minimizing the total shaft power of compressors. The objective model is shown in Eq. (27), where W is the total shaft power of compressors, and $W_{m',m}$ indicates shaft work of compressors between level m' and m .

$$\min W = \sum_{m'=m+1}^M \sum_{m=0}^M W_{m',m} \quad \forall m, m' \in SM, m < m' \quad (27)$$

The method of calculating $W_{m',m}$ is given in Eq. (28), where $wf_{m',m}$ represents the specific shaft work of compressors from level m' to level m . It can be obtained by rigorous simulation, when compressor efficiency is supposed to be 72%.

$$W_{m',m} = f_{c_{m',m}} wf_{m',m} \quad \forall m, m' \in SM, m < m' \quad (28)$$

2.2.5. Equality constraint

The binary variable y_m values equal one when the pressure of m^{th} level exists; otherwise it equals zero. The condenser in the refrigeration system is considered to be the highest pressure level in the whole system at level 0. Since the 0-stage condenser is constant, y_0 equals one correspondingly, as shown in Eq. (29).

$$y_0 = 1 \quad (29)$$

Assuming that the total mass flow rate of the refrigerant in the refrigeration system is f_0 , the outlet flow rate of the 0th compressor, fe_0 , is equal to f_0 , and in order to ensure the system closed, there is no liquid discharge for the flash tank at level M .

$$\sum_{m=1}^M f_{c_{m,0}} = f_0 \quad (30)$$

$$fe_0 = f_0 \quad (31)$$

$$fe_M = 0 \quad (32)$$

In this optimization problem, the refrigerant in the evaporators is regarded as cold stream, while it is considered as hot flow in the sub-cooling sections. So there are no hot or cold utilities in this system, which means, the remaining heat equals zero at 0th and M^{th} intervals.

$$R_0 = 0 \quad (33)$$

$$R_j = 0 \quad (34)$$

2.2.6. Inequality constraint

The binary variables y_m is used to logically determine the existence of the m^{th} level. The corresponding variables equal zero when the m^{th} level does not exist. Besides, Z is a suitable large number.

$$f_{c_{m',m}} \leq y_m Z \quad \forall m, m' \in SM, m < m' \quad (35)$$

$$f_{sc_{m',m}} \leq y_m Z \quad \forall m, m'' \in SM, m' < m, m > 0 \quad (36)$$

$$Qx_m \leq y_m Z \quad \forall m, m' \in SM, m > 0 \quad (37)$$

What's more, the total number of levels should be limited according to the specific problem.

$$\sum_{m=1}^M y_m \leq M \quad (38)$$

Finally, on the basis of the Second Law of Thermodynamics, the remaining heat of each interval should be nonnegative variables. Besides, some variables should be nonnegative according to the practical production.

$$R_j \geq 0 \quad \forall j \in SJ \quad (39)$$

$$fg_m, fe_m, fx_m, fv_m, fve_m, Qx_m \geq 0 \quad \forall m \in SM, m > 0 \quad (40)$$

$$fc_{m',m}, fsc_{m',m}, vf_{m,m'} \geq 0 \quad \forall m \in SM, m > 0 \quad (41)$$

2.2.7. Summary of the mathematical model

The total model can be summarized as follows:

$$\text{Min } W = \sum_{m'=m+1}^M \sum_{m=0}^M W_{m',m} \quad (27)$$

s.t. Mass balance (1–8)

Energy balance (10)

Equality constraint

(12, 13, 18–20, 22–25, 35–41)

Inequality constraint

(11, 21, 26, 28–34)

$y_m, y_{j,m',m}^1, y_{j,m',m}^2$ and $y_{j,m',m}^3$ are integer variables. The MINLP model is established to optimize the refrigeration system. There are

eighteen variables in this model, including four binary variables, eight nonnegative variables about mass flow rate, three variables representing thermal load, two variables in compressing section and one temperature variable $T_{sc,m,m}$. Other parameters will be given as known data according to the specific optimization process. GAMS software will be used to solve the model, with the BARON solver and solving method of MINLP.

3. Case Study

3.1. Base case

The optimization of the ethylene refrigeration system in an existing ethylene plant is studied to demonstrate the proposed MINLP model. The original system provides cold duties for four process streams, whose temperature ranges from $-47\text{ }^{\circ}\text{C}$ to $-97.9\text{ }^{\circ}\text{C}$. Fig. 6 gives the basic structure of the cascade refrigeration system to be optimized.

As is shown in Fig. 6, the initial refrigeration cycle consists of four pressure levels, and the highest pressure is 1645 kPa, which is designated as the 0th level. Apart from the condenser, there are four evaporators and three sub-coolers in this closed system, and the information of Process streams through the heat exchangers is listed in Table 1. There are five cold streams and four hot streams, so the relevant L is equal to five, and N is equal to four. The refrigeration system should meet the demand of the cooling quantity of process flows.

According to the rigorous simulation of the total system, the Exergy–Temperature plot is given in Fig. 7. Exergy is defined as the maximum amount of useful energy from a process needed in order to bring a system to equilibrium with its surrounding environment, and it is denoted as B in this paper [10]. A B - T plot consists of saturated liquid line (plotted in red), saturated vapor line (plotted in blue) and closed-loop working line of the refrigerant, which is assumed as pure ethylene.

In this chart, the working lines express the physical property of the refrigerant in the refrigeration system. As shown by C-1, the vapor phase refrigerant is compressed after passing through the compressor C-1, the exergy and temperature of the refrigerant increase obviously. Similarly, C-2 and C-3 correspond to the physical property of the refrigerant in other two compressors. Then, the superheated steam passes through a condenser, transferring heat to a high-temperature sink. The temperature drops and the refrigerant becomes the saturated liquid. So the working line falls on the saturated liquid line. E313XA,

Table 1

Given Information of process streams served by the ethylene refrigeration system

Stream ID	Related unit ID	RELATED UNIT TYPE	Inlet $T/^{\circ}\text{C}$	Outlet $T/^{\circ}\text{C}$	$Q/\text{kJ}\cdot\text{h}^{-1}$
1	CO-1	Cooler	-56.1	-55.7	8964000
2	CO-2	Cooler	-81	-40	2498040
3	CO-3	Cooler	-81	-40	353880
4	CO-4	Cooler	-81	-40	541080
5	CO-5	Cooler	-81	-40	1283760
6	E-1	Evaporator	-47	-57	13974840
7	E-2	Evaporator	-57	-77	16691040
8	E-3	Evaporator	-87	-98	4595760
9	E-4	Evaporator	-96.2	-97.9	6590160

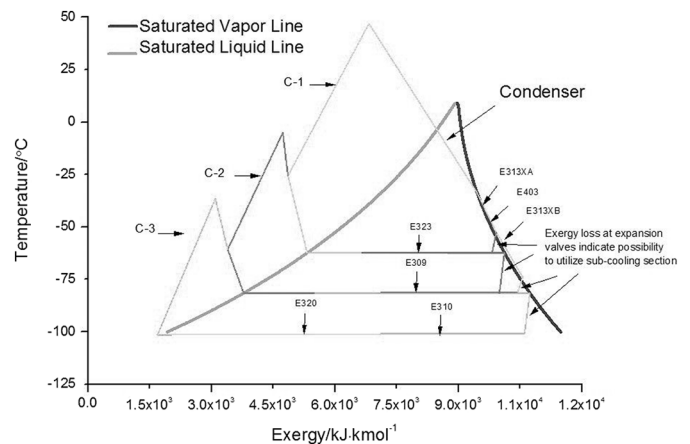


Fig. 7. B - T plot of the original system.

E403 and E313XB show temperature reduction of refrigerant leaving sub-coolers, and the working line changes along the saturated liquid line. After it goes through the expansion valve, both temperature and exergy decrease obviously, meanwhile, the liquid phase refrigerant is partially gasified. Next, the refrigerant is isothermally vaporized while gaining heat when it passes through an evaporator, like E323, E309, E310 and E320. The working line is kept horizontal and shifted to the saturated vapor line. After that, it can be divided into saturated vapor and liquid through a flash drum. Then the vapor steam is compressed

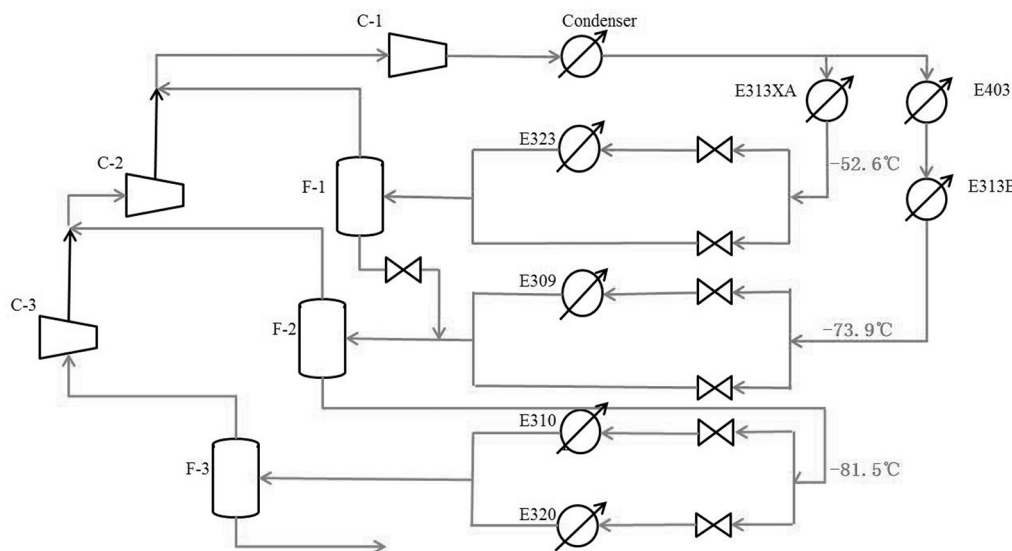


Fig. 6. Diagrammatic sketch of the original ethylene refrigeration system.

to the high level, while the liquid steam goes to the low level through the sub-cooler and expansion valve. It can be concluded from this figure that exergy loss in this system mainly focus on the expansion valves, and the use of sub-coolers can significantly reduce the loss in the expansion section. Accordingly, reasonable allocation of pressure levels and effective utilization of sub-coolers are important for an optimal system.

3.2. Optimal retrofit solution

The cooling system has four original pressure levels (level 0, 2, 4 and 6). Three candidate stages (1, 3 and 5) are added, obtaining the pressure set, $SM = \{0, 1, 2, 3, 4, 5, 6\}$. The temperature at each level is the bubble temperature of ethylene at the corresponding pressure, as shown in Table 2. Besides, the minimum approach temperature is 2 °C. The MINLP model was formulated in GAMS and optimized by the BARON solver to obtain the global optimal solution.

Table 2

Candidate pressure levels employed in the optimization model

m	0	1	2	3	4	5	6
$P_m/\text{kPa}\cdot\text{g}$	1645	1117.5	590	403	216	114.5	13
$T_m/^\circ\text{C}$	−34	−45.7	−62.5	−70.6	−81.5	−89.8	−101.5

In light of the calculation result, the simple structure of the optimized ethylene cascade refrigeration system is shown in Fig. 8, and the detailed information of the optimal case is summarized in Tables 3 and 4.

Due to the application of the HEN Synthesis transshipment model, the hot and cold process streams were better matched, which resulted in less demand for refrigerant. And the locations of the sub-coolers were also changed. What's more, the outlet temperature after the sub-coolers was determined with the GDP method. For example, comparing Figs. 6 and 8, in the optimal solution, the temperatures were −62.23 °C after sub-cooler1, −79 °C after sub-cooler2, and −87 °C after sub-cooler3. They were all lower than before. Therefore, the working lines of the expansion valves in B - T plot are shorter, which led to less exergy loss in this system.

According to Table 5, it can be seen that the total shaft work in the optimal case was 4.521 MW, which was reduced by 15.6% compared with the base case. After rigorous simulation by Aspen Plus,

Table 3

Stream information of the optimal C2 refrigeration system

m	m'	$f_{sc,m,m'}/\text{kg}\cdot\text{h}^{-1}$	$T_{sc,m,m'}/^\circ\text{C}$	$Q_{f,m,m'}/\text{kJ}\cdot\text{h}^{-1}$
0	2	86789.6	−62.225	6.22×10^6
2	4	55208.1	−79	2.31×10^6
4	6	26376.4	−87	3.68×10^5

Table 4

Distribution of refrigerant flow rates and cooling duties in each evaporation section of the optimal system

m	$f_{g,m}/\text{kg}\cdot\text{h}^{-1}$	$f_{e,m}/\text{kg}\cdot\text{h}^{-1}$	$Q_{X,m}/\text{kJ}\cdot\text{h}^{-1}$
0	–	86789.615	–
2	31581.467	55208.148	1.30×10^7
4	28831.772	26376.375	1.25×10^7
6	26376.375	–	1.16×10^7

Table 5

Comparison between the base and optimal cases of the C2 refrigeration system

Compressor	Base case		optimal case	
	Level m	$W_{m',m}/\text{MW}$	Level m	$W_{m',m}/\text{MW}$
C-1	2	3.346	2	2.735
C-2	4	1.398	4	1.161
C-3	6	0.615	6	0.624
W		5.359		4.521
Reduction/%				15.6

the feasibility of the optimal flowsheet was verified. The Exergy–Temperature plot of the optimal cascade system was shown in Fig. 9. Comparing Figs. 9 and 7, the sub-cooling line of the optimization case stays closer to the saturation liquid line and the two long expansion lines are shorter, signifying less exergy loss. The result means that the optimal case is more reasonable for the allocation of sub-coolers than the base case, and the exergy loss of expansion valve is reduced.

In sum, the optimal system requires less compressor shaft work and shows better refrigeration performance. Therefore, the optimization method proposed in this paper is feasible and can be applied in more refrigeration processes.

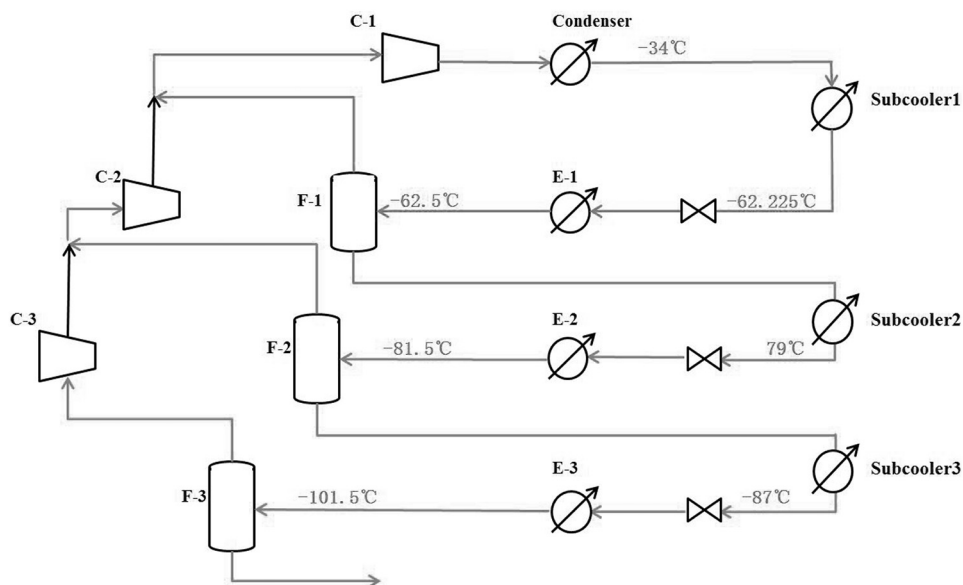


Fig. 8. Design of the ethylene refrigerant cycle from the optimal solution.

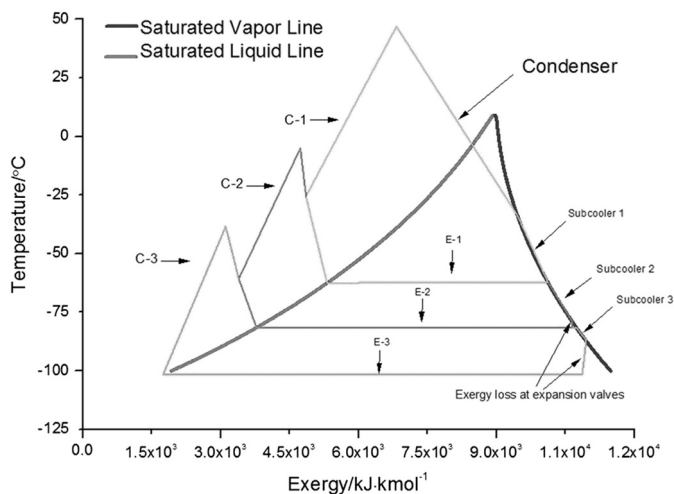


Fig. 9. B-T plot of the optimized system.

4. Conclusions

The optimal synthesis of refrigeration system is a significant but challenging problem to chemical/petrochemical process industries. In this paper, a general framework was established to optimize the refrigeration system and HEN simultaneously. A new superstructure of refrigeration system was raised, in which the refrigerant between levels passed through an expansion valve after a sub-cooler. In this way, the exergy loss of the refrigeration system can be reduced because of the rational utilization of sub-coolers. However, it led to branching selections, and the GDP method was introduced to solve the problem. The GDP model was then reformulated as an MINLP model, with the objective of minimizing the total compressor shaft work, by Big-M Constraint Method.

Besides, a case study on the retrofit design of an ethylene refrigeration system in an existing ethylene plant demonstrated the effectiveness of the optimization framework. A multi-level refrigeration system satisfying all of cooling and heating demands was obtained by solving the MINLP model, and the result showed that the total compressor shaft work was reduced by 15.6% compared to the original system. Finally, rigorous simulation was made and the feasibility and consistency of the optimal solution was verified.

However, much more efforts and studies are required. In this paper, the objective is minimizing the total compressor shaft work. Obviously, as the pressure levels increase, the total shaft work is reduced, but the equipment investment rises. Therefore, the total annual cost is more reasonable as the objective function. And we are currently working on it.

Nomenclature

C_p	average heat capacity of the refrigerant, $\text{kJ} \cdot \text{kg}^{-1} \cdot ^\circ\text{C}^{-1}$
f	total mass flow rate, $\text{kg} \cdot \text{h}^{-1}$
f_c	outlet flow rate of the compressor, $\text{kg} \cdot \text{h}^{-1}$
f_e	mass flow rate of the saturated liquid, $\text{kg} \cdot \text{h}^{-1}$
f_g	mass flow rate of the saturated vapor, $\text{kg} \cdot \text{h}^{-1}$
f_{sc}	mass flow rate through the sub-coolers, $\text{kg} \cdot \text{h}^{-1}$
f_v	mass flow rate of saturated vapor generated by the valves, $\text{kg} \cdot \text{h}^{-1}$
f_{ve}	mass flow rate of saturated vapor generated by the evaporating section, $\text{kg} \cdot \text{h}^{-1}$
f_x	inlet mass flow rate to flash drum, $\text{kg} \cdot \text{h}^{-1}$
HF	mass enthalpy at the entrance of the expansion, $\text{kJ} \cdot \text{kg}^{-1}$

HL	liquid enthalpy at the outlet of the expansion, $\text{kJ} \cdot \text{kg}^{-1}$
HV	gas enthalpy at the outlet of the expansion, $\text{kJ} \cdot \text{kg}^{-1}$
M_1	a reasonable large number from GDP model
M_2	a reasonable large number from GDP model
P	pressure of refrigerant, kPa-g
PH	set of the heat process streams
PL	set of the cold process streams
QC	heat duty of cold streams, $\text{kJ} \cdot \text{h}^{-1}$
QH	heat duty of hot streams, $\text{kJ} \cdot \text{h}^{-1}$
Q_t	heat duty of sub-coolers, $\text{kJ} \cdot \text{h}^{-1}$
Q_x	heat duty of evaporators, $\text{kJ} \cdot \text{h}^{-1}$
R	remaining heat, $\text{kJ} \cdot \text{h}^{-1}$
SM	set of the total number of refrigeration levels
SJ	set of the temperature interval in the HEN
T	saturated temperature at each pressure level, $^\circ\text{C}$
Te	temperature at each interval, $^\circ\text{C}$
T_{sc}	temperature after sub-coolers, $^\circ\text{C}$
vf	the vapor fraction at the expansion valve
W	compression shaft work, $\text{W} \cdot \text{kg}^{-1} \cdot \text{h}^{-1}$
wf	the specific shaft work of compressors, $\text{W} \cdot \text{kg}^{-1} \cdot \text{h}^{-1}$
xi	a binary index vector
Y	a logical variable from GDP model
y	a binary variable
Z	a reasonable large number
z	a binary index vector
Δhr	latent heat of vaporization, $\text{kJ} \cdot \text{kg}^{-1}$

Subscripts

h	index for heat process streams ($h = 1, 2, \dots, H$)
j	index for the temperature interval ($j = 1, 2, \dots, J$)
l	index for cold process streams ($l = 1, 2, \dots, L$)
m	index for the potential refrigeration level ($m = 1, 2, \dots, M$)

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